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On vortex shedding from smooth and rough cylinders in the range of Reynolds numbers 6×10^3 to 5×10^6

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The influence of surface roughness on the vortex-shedding frequency in the wake of a single cylinder has been investigated. The experiments were carried out in an atmospherical and a high-pressure wind tunnel. The tests were started with a smooth cylinder. Then the wake flow of cylinders with relative roughnesses of $k_s/d = 75 \times 10^{-5}$, 300×10^{-5}, 900×10^{-5}, and 3000×10^{-5} was investigated.

For all roughness parameters tested the Strouhal number exhibited an increase in the critical flow regime. With growing roughness parameter the step in the curve became smaller. At transcritical flow conditions the Strouhal number was measured to be in the range of $Sr = 0{\cdot}25 \pm 0{\cdot}018$ for all surface roughness tested. No regular vortex shedding could be observed in the critical flow range for the smooth cylinder with $l/d = 3{\cdot}38$. When prolonging the test body to $l/d = 6{\cdot}75$ the wake fluctuations became periodic.

1. Introduction

From recent experiments it is known (Groehn & Scholz 1977) that the heat transfer of heat exchangers with tubes in cross flow can be enhanced considerably without increase in pressure drop by applying artificially roughened tubes. However, flow fluctuations in tube banks may lead to mechanical or acoustical resonance effects which in turn cause mechanical damage to the heat-exchanger components (Heinecke, 1971). Before starting investigations on vortex shedding of multi-tube arrays the non-steady flow phenomena of rough single cylinders were studied. Those experiments require the ability to produce very high Reynolds numbers. They could be realized by using a high-pressure wind tunnel and by conceding a tunnel blockage ratio of $d/b = 1/6$ and a length to diameter ratio of $l/d = 3{\cdot}38$, as well.

The effect of blockage ratio on the flow past bluff bodies has been considered previously by numerous authors. At least for subcritical or transcritical flow conditions it can be accounted for by applying the formulae of Allen & Vincenti (1930), for instance.

The influence of the aspect ratio l/d on the flow past a smooth cylinder has been treated by Morsbach (1967). He found that the flow is most affected in the critical flow regime. For $l/d \leqslant 3$ the separation line of the boundary layer is no longer parallel to the cylinder axis, which means that the flow is three-dimensional. With regard to Morsbach's results the authors expected an effect on the vortex-shedding mechanism due to the low aspect ratio for the smooth cylinder at critical flow conditions. Therefore an additional test run was carried out using an aspect ratio of

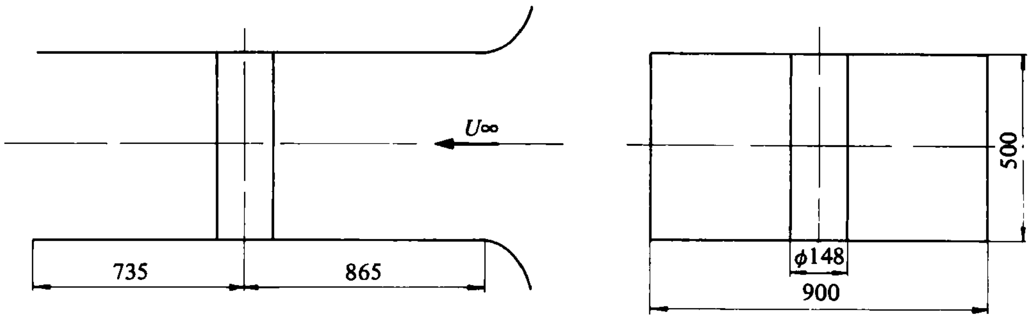

FIGURE 1. Test section, dimensions in millimetres.

$l/d = 6{\cdot}75$. For rough surface cylinders an aspect ratio of $l/d = 3{\cdot}38$ was sufficient to obtain two-dimensional flow conditions for the whole Reynolds-number range investigated. This results from a check carried out for the coarsest cylinder and for that endued with the lowest surface roughness. The vortex-shedding frequencies at $l/d = 3{\cdot}38$ and $l/d = 6{\cdot}75$ were the same for the particular roughness conditions.

To make sure that the experimental set-up and the measuring technique were not the source of systematic errors a smooth cylinder was taken as the first test object. Comparison of the results so obtained with those of other authors (Bearman 1969; Roshko 1961; Delany & Sorensen 1953; Jones, Cincotta & Walker 1969) indicated that the experimental procedure was accurate.

2. Experimental arrangement

The test object was mounted vertically in a rectangular test section. The dimensions are given in figure 1. The whole array could be transferred from the atmospheric to the high-pressure wind tunnel (maximum pressure 40 bar), because both had the same geometrical dimensions. The range of Reynolds number which could be covered with the given arrangement was $1{\cdot}5 \times 10^4$–3×10^5 (atmospheric tunnel) and $1{\cdot}3 \times 10^5$–5×10^6 (high-pressure tunnel). The turbulence level of the incident flow was $Tu = 0{\cdot}45\,\%$ in both wind tunnels. A detailed description of the test facilities has been given by Grosse & Scholz (1965). The roughest cylinder ($k_s/d = 3000 \times 10^{-5}$) was tested in an additional open wind tunnel using a test cross-section of 500 mm square and a cylinder diameter of $d = 25$ mm.

The smooth cylinder ($d = 148$ mm) was made of highly polished copper. The relative roughness was $k/d < 10^{-5}$, where k means the total height of the surface roughness measured by means of a roughness meter. The roughness elements of the other cylinders were produced by a knurling process. By this method pyramids of rhombic basic area and definite heights were formed and a uniform roughness on the whole surface of each cylinder could be achieved. The geometrical data of the roughness pattern are given in figure 2. The effective diameter used in the dimensionless groups is taken as that of the bare cylinder (roughness elements ground off) plus the height of a roughness element.

The classification of the pyramidal surface roughness has been a severe problem. Since a roughness pattern is characterized not only by the height of the elements, but also by their local arrangement on the surface and by the statistical distribution

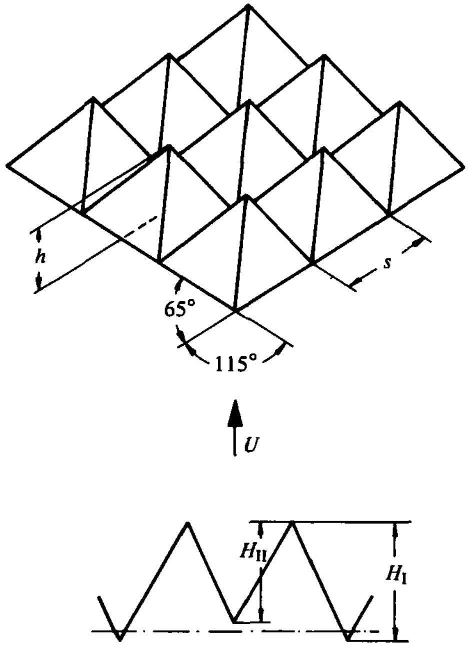

FIGURE 2. Roughness pattern.

H_I (mm)	0·10–0·12	0·47	0·90–0·92	0·75–0·825
H_{II} (mm)	0·05–0·07	0·4	0·7	0·60–0·675
s (mm)	0·31	7·2	2·1	2·1
k_s/d	75×10^{-5}	300×10^{-5}	900×10^{-5}	3000×10^{-5}

Table 1

of the different grain sizes, it seemed unsatisfactory to the authors to report only on the geometric size of the pyramidal roughness. Therefore further information is given by quoting its effect on the flow. A measure to do so is to consider the critical Reynolds numbers as a function of surface roughness.

Fage & Warsap (1930), Achenbach (1971) and Miller (1976) have published drag-coefficient data of rough cylinders which exhibit a relationship between the dimensionless equivalent sand-grain roughness k_s/d and the critical Reynolds number Re_{crit}. This relationship can be approximated by

$$Re_{\text{crit}} = \frac{6000}{(k_s/d)^{\frac{1}{2}}}.$$

Introducing the values of the critical Reynolds number into this equation the equivalent sand-grain roughness can be determined for the particular pyramidal surface roughnesses. The corresponding result is given in table 1. The application of this method is not free of uncertainties, but it represents the best one available to the authors at this time.

For measurement of the flow fluctuations a hot-wire probe was installed 0·11 m

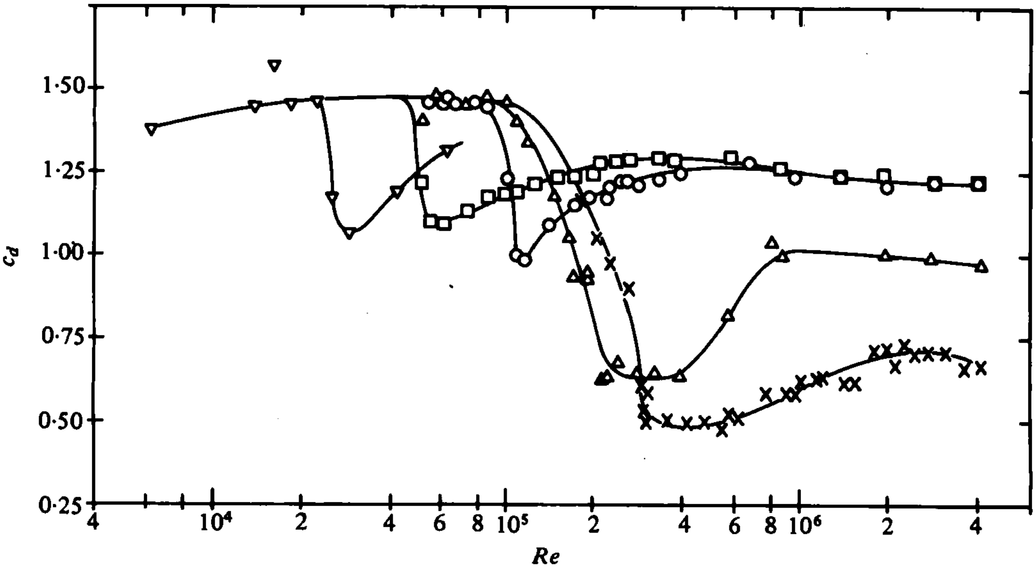

FIGURE 3. Drag coefficient of the single cylinder in cross flow at various surface roughness parameters k_s/d: ×, smooth; △, 75×10^{-5}; ○, 300×10^{-5}; □, 900×10^{-5}; ▽, 3000×10^{-5}.

behind the rear stagnation point of the cylinder. The spanwise direction of the wire was parallel to the axis of the cylinder. The probe shaft could be traversed through the wake. The position of the hot wire was detected by a precision resistance potentiometer which was fastened to the moving slide where the probe shaft was mounted. At the probe shaft an accelerometer was installed to measure the mechanical oscillations of the hot-wire probe due to wind forces. In this way it was demonstrated that the frequencies indicated were not caused by vibrations of the hot-wire probe itself.

The hot-wire probe was connected to a CTA-bridge (DISA, 55 Mo 1). The linearized signal was analysed by a heterodyne analyser (Brüel–Kjaer 2010) and the spectral distribution was registered with a level recorder. In addition the signal was observed by means of an oscilloscope. This was helpful in finding the optimum position of the hot wire in the wake.

3. Results

Before discussing the measurements of the non-steady wake flow some remarks on the influence of roughness on the steady flow field around the cylinder should be made. A detailed description of the effects observed has been given by Achenbach (1971). In those tests he used tubes the surfaces of which were clad with emery paper, whereas the present experiments were performed with artificially roughened cylinders. This technique was applied because the cylinders were also used for heat-transfer experiments. From measurements of the static pressure around the circumference of the artificially roughened tubes the drag coefficient had been calculated by Achenbach (1977) previously. The results, not corrected for blockage effects, are given in figure 3.

Comparison between the drag-coefficient curves of the pyramidal and the emery-paper roughness reveals some discrepancy concerning the absolute value of the drag coefficient. For subcritical flow conditions the deviation is about 10%. Owing to the large diameter of the cylinder the velocities and thus the pressure difference around the cylinder were very low in this Reynolds-number range. As the drag ooefficient was calculated from the local static pressure distribution the difference may result from this deficiency. Additionally it should be realized that the type of the roughness was completely different in the two investigations, which also may have had an effect on the drag. The agreement of the drag coefficient results for the two roughness types is very good in the transcritical flow range. In the critical flow regime, which is most sensitive to disturbances, there are discrepancies which are most probably due to the fact that two roughness types, the first a regular arrangement of equal roughness elements, the second consisting of different fractions of granulate, were used.

In spite of these discrepancies the fundamental boundary-layer phenomena are the same for both roughness types. In the subcritical range where the boundary layer is laminar throughout roughness has no influence on the drag. The critical flow regime is indicated by the drop of the drag coefficient due to the rearward shift of the laminar boundary-layer separation point. At the critical Reynolds number, which is the Reynolds number at c_d minimum, laminar intermediate separation and turbulent reattachment occurs. The turbulent boundary layer transferring a higher amount of energy normal to the wall than the laminar boundary layer enables the flow to follow the contour of the cylinder far downstream. The value of the critical Reynolds number decreases with increasing roughness parameter. At the same time the critical-Reynolds-number range is shrinking. In the supercritical regime c_d grows again. Intermediate laminar separation is no longer observed, but an immediate transition from laminar to turbulent flow. The location of transition shifts upstream with increasing Reynolds number.

Finally the transcritical flow range is reached where c_d is almost constant. The boundary layer undergoes transition from laminar to turbulent flow in the vicinity of the front stagnation point. With increasing relative roughness c_d increases, then levelling out to $c_d \sim 1{\cdot}2$ for $k_s/d \geqslant 300\times 10^{-5}$. This is not evident for the coarsest cylinder ($k_s/d = 3000\times 10^{-5}$), which, through lack of time, could not be tested in the high-pressure wind tunnel at high Reynolds numbers. The drag-coefficient data of this cylinder presented in figure 3 are extrapolated for a blockage ratio of $d/b = 1/6$ according to the formulae of Allen & Vincenti (1930).

In figure 4 the Strouhal number Sr is plotted against the Reynolds number Re for the smooth cylinder. In the subcritical regime Sr is almost constant, $Sr = 0{\cdot}205$. Beyond the critical Reynolds number no regular signals were found in the wake of the cylinder with $l/d = 3{\cdot}38$, though the hot-wire probe was traversed ± 1 diameter from the wake centre-line across the wake at different distances downstream of the cylinder. This phenomenon was supposed to be due to three-dimensional wake formation as mentioned above. Tests with a prolonged cylinder ($l/d = 6{\cdot}75$ and a blockage ratio of $d/b = 1/6$) showed that also in the critical flow regime quasi-regular flow fluctuations occurred. The Strouhal number was around $Sr = 0{\cdot}5$. Bearman (1969) measured a lower value of $Sr = 0{\cdot}46$ because he carried out his experiments using a smaller blockage ratio of $d/b = 1/15{\cdot}4$. His observation was confirmed that between the subcritical Strouhal number $Sr = 0{\cdot}2$ and the critical value of $Sr = 0{\cdot}5$ only two

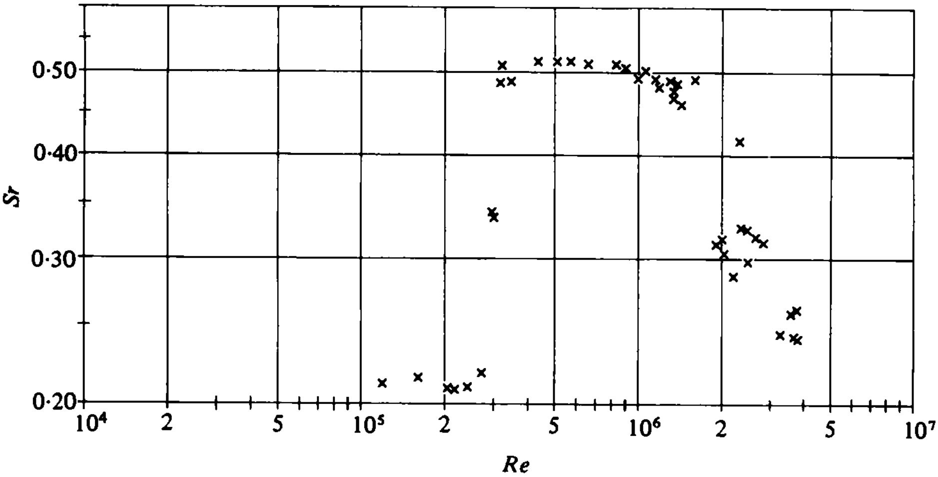

FIGURE 4. Smooth cylinder (l/d = 6·75; d/b = 1/6): Strouhal number *vs.* Reynolds number.

intermediate values exist at Sr = 0·34. According to Bearman's findings this phenomenon appears to be due to the occurrence of a separation bubble on one side only of the cylinder.

The values of Sr presented in figure 4 have not been corrected for tunnel blockage in the total Reynolds-number range investigated since beyond the critical Reynolds number the angle of boundary-layer separation is $\phi_s > 90°$, for which the formulae of Allen & Vincenti (1930) cannot be applied.

The supercritical flow range ends at about $Re = 2 \times 10^6$. The Strouhal number decreases stepwise from Sr = 0·5 at $Re = 1{\cdot}5 \times 10^6$ down to a value of Sr = 0·25 at $Re = 4 \times 10^6$. The fluctuation spectrum was broad compared with that at the lower Reynolds numbers. Thus the determination of the vortex-shedding frequency was subject to a larger scatter band.

In figure 5 results for $k_s/d = 75 \times 10^{-5}$ are plotted. Sr is nearly constant in the subcritical range, which ends at about $Re = 1{\cdot}5 \times 10^5$. In the critical and supercritical flow range, high and low values of Sr (0·5–0·18) are found. Obviously the relative roughness is still too small to stabilize the vortex-shedding mechanism in such a way as is observed for higher Reynolds numbers or higher roughnesses. Since the flow is sensitive to disturbances, the character of the flow around the cylinder may randomly be critical or subcritical, each range being characterized by different values of Strouhal number. This phenomenon was observed for both aspect ratios l/d = 3·38 and l/d = 6·75. Beyond $Re = 5 \times 10^5$ the transcritical flow range is reached, where the flow is stabilized and the Strouhal number is only weakly dependent on Reynolds number.

Whereas for the smooth cylinder and that with the smallest roughness parameter the critical flow regime is extended over a certain range of Reynolds number, this is no longer the case for higher roughness parameters. As can be seen in figures 6, 7, and 8 the transition from subcritical to transcritical values of Strouhal number occurs within a small range of Reynolds number. Referring to the drag coefficient (figure 3) this range corresponds to that of the intermediate decrease of c_d. With increasing roughness the minimum of c_d at the critical Reynolds number increases,

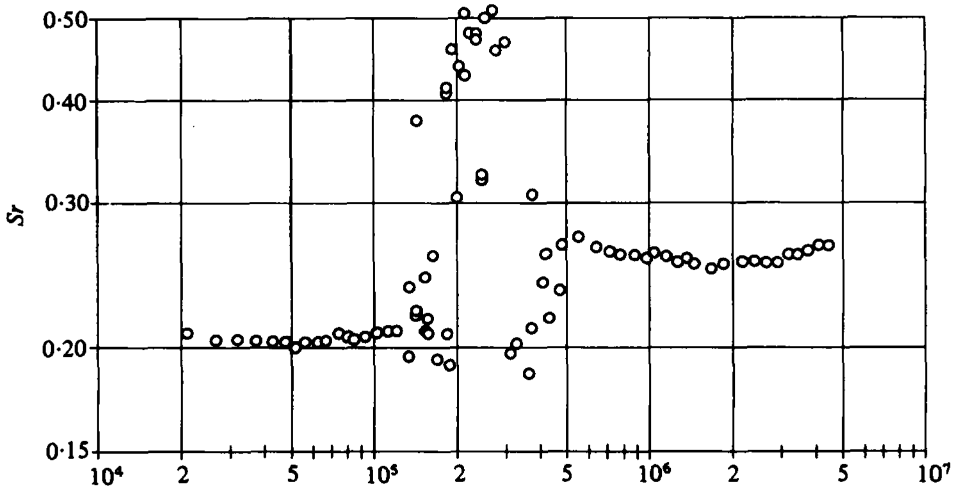

FIGURE 5. Rough cylinder $k_s/d = 75 \times 10^{-5}$; $l/d = 6{\cdot}75$; $d/b = 1/6$): Strouhal number *vs.* Reynolds number.

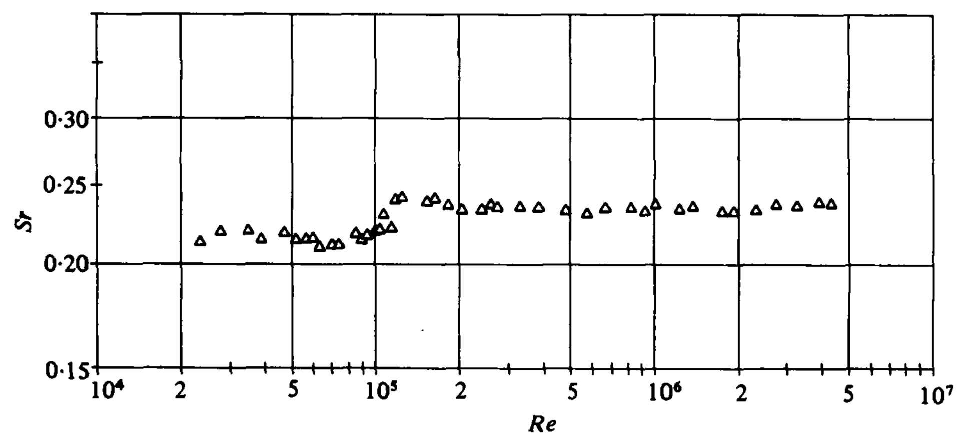

FIGURE 6. Rough cylinder $k_s/d = 300 \times 10^{-5}$; $l/d = 3{\cdot}38$; $d/b = 1/6$): Strouhal number *vs.* Reynolds number.

whereas the Strouhal number in the corresponding flow range becomes smaller for rough than for smooth cylinders (see figure 9). Both phenomena are associated with the shifting of the location of the boundary-layer separation. With increasing roughness parameter the angle of separation, ϕ_s, becomes smaller ($\phi_s \sim 110°$) compared with that of the smooth cylinder ($\phi_s = 140°$) at critical flow conditions (Achenbach 1971). The major change occurs in the range $75 \times 10^{-5} < k_s/d < 300 \times 10^{-5}$ in which, therefore, the frequency of vortex formation also varies most considerably as a function of the roughness parameter.

In the transcritical region the angle of boundary-layer separation ϕ_s varies between $90° < \phi_s < 110°$ for different roughnesses (Achenbach 1971). Since the wake width does not change remarkably when ϕ_s is varied in this range, a strong variation of Strouhal number will not occur (see figure 9). Nevertheless, the reader would expect

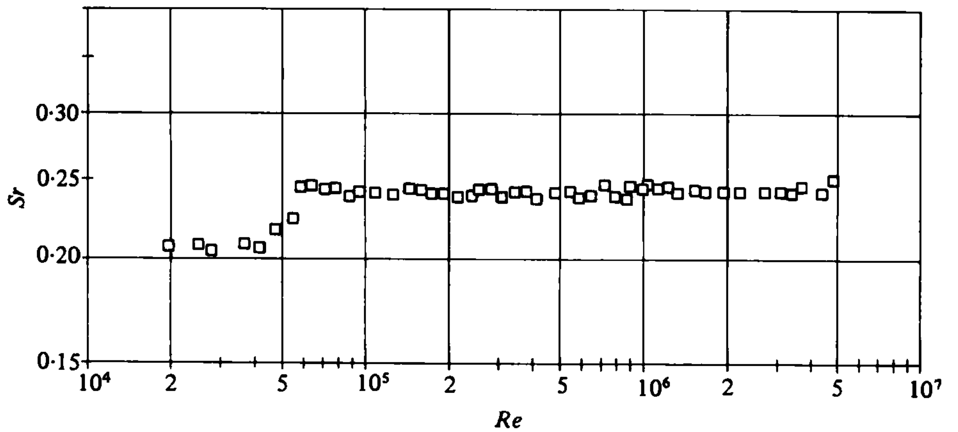

FIGURE 7. Rough cylinder ($k_s/d = 900 \times 10^{-5}$; $l/d = 3{\cdot}38$; $d/b = 1/6$): Strouhal number *vs.* Reynolds number.

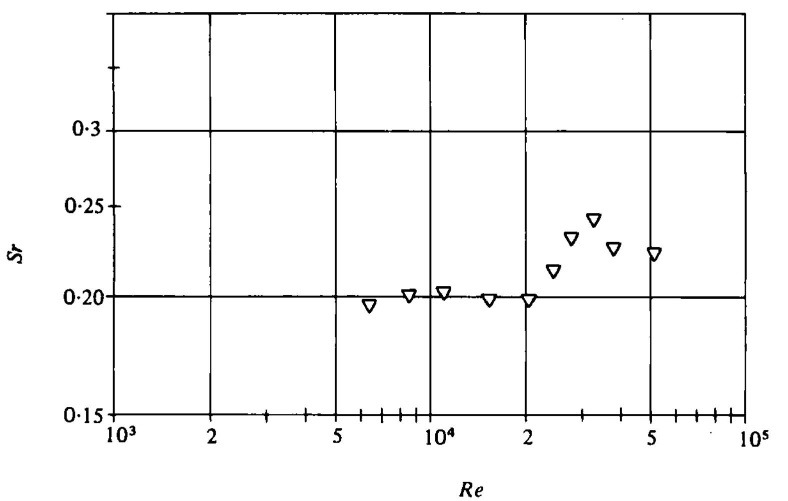

FIGURE 8. Rough cylinder ($k_s/d = 3000 \times 10^{-5}$; $l/d = 20$; $d/b = 1/20$): Strouhal number *vs.* Reynolds number.

a systematic trend of the results in such a way that the transcritical Strouhal number decreases with increasing roughness parameter. This, however, could not be confirmed in the present experiments. As the total variation of the transcritical Strouhal number with the roughness parameter is smaller than $\pm 7\,\%$ of the mean value, the expected effect is lost in the scatter band of the experimental results.

For all rough cylinders the flow fluctuations had narrow band character in the whole range examined. The only exception was the cylinder with the smallest surface roughness parameter $k_s/D = 75 \times 10^{-5}$, which showed intermittency effects in the critical flow regime as described above.

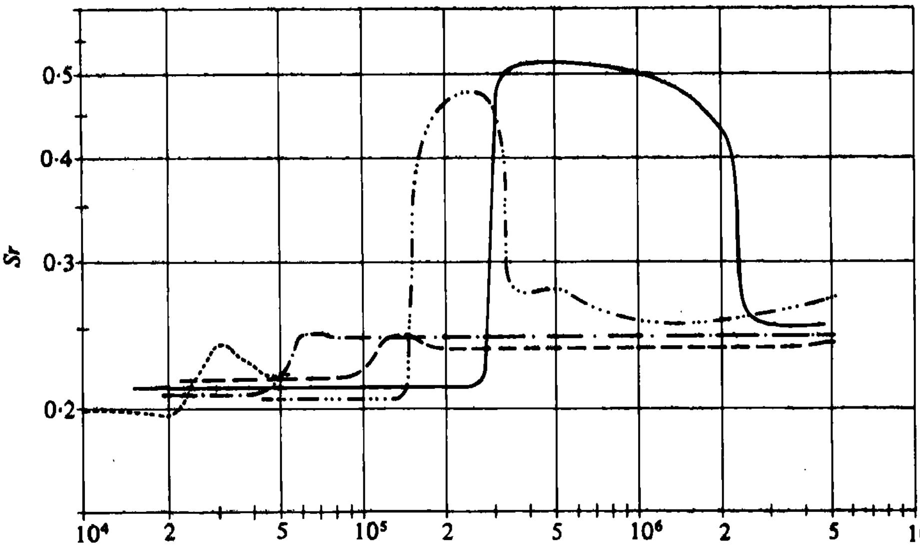

FIGURE 9. Circular cylinder: Strouhal number *vs.* Reynolds number at variable surface roughness conditions. ——, smooth; —··;—, $k_s/d = 75\times10^{-5}$; — —, $k_s/d = 300\times10^{-5}$; —·—, $k_s/d = 900\times10^{-5}$; ----, $k_s/d = 3000\times10^{-5}$.

4. Discussion and comparison with other authors

4.1. *Smooth cylinder*

In the subcritical range the results are quite near to those published by many other authors previously. As representative results those of Roshko (1961) and Drescher (1956) are plotted in figure 10. The rapid increase of the Strouhal number at critical flow conditions has been observed by several authors. Delany & Sorensen (1953) found a sudden decrease of their high values of Sr at $Re = 2\times10^6$, which indicates the transition to transcritical flow conditions. A similar result was found in the present work for an aspect ratio of $l/d = 6{\cdot}75$. The results of Bearman (1969) have already been discussed above.

In the transcritical range the Strouhal number had a value of $Sr = 0{\cdot}25$, which is about 10 % lower than the value reported by Roshko (1961). Ruscheweyh (1974) measured separation frequencies at a television tower in natural wind for $6\times10^6 < Re < 1{\cdot}4\times10^7$. He found the value of $Sr = 0{\cdot}235$. It must, however, be remarked that Ruscheweyh's test object was not smooth. Owing to irregularities of the concrete surface the maximum roughness elements were of the order of 11 mm (diameter of the tower: 10·14 m). Thus the relative roughness was about $k/d = 100\times10^{-5}$. However, the roughness was not uniformly distributed over the surface. Since there is no information on the effect of individually distributed roughness on the Strouhal number in transcritical flow conditions it cannot be concluded that Ruscheweyh's low value is due to roughness effects.

The highest value of transcritical Strouhal number was measured by Jones *et al.* (1969) in a transonic wind tunnel. In those tests the Mach number defined as the ratio of incident velocity, U_∞, to the velocity of sound, a, had values up to $Ma = 0{\cdot}46$

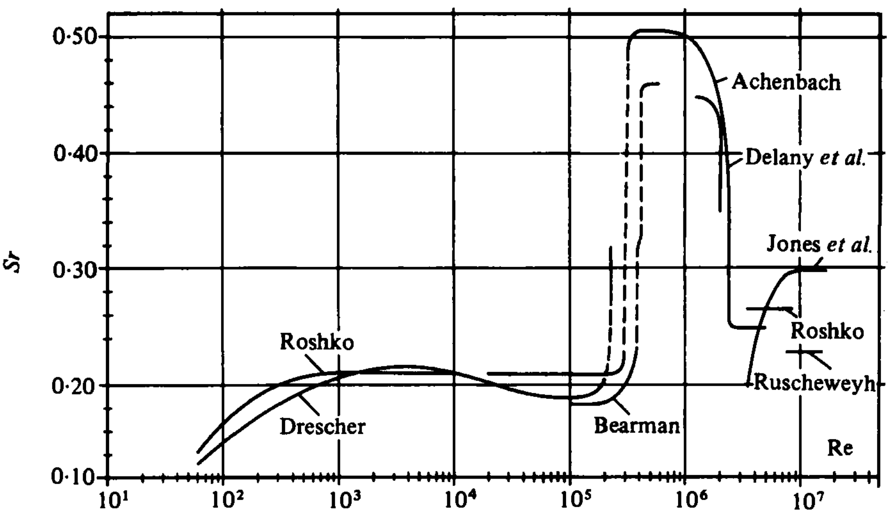

FIGURE 10. Smooth circular cylinder: Strouhal number *vs*. Reynolds number.

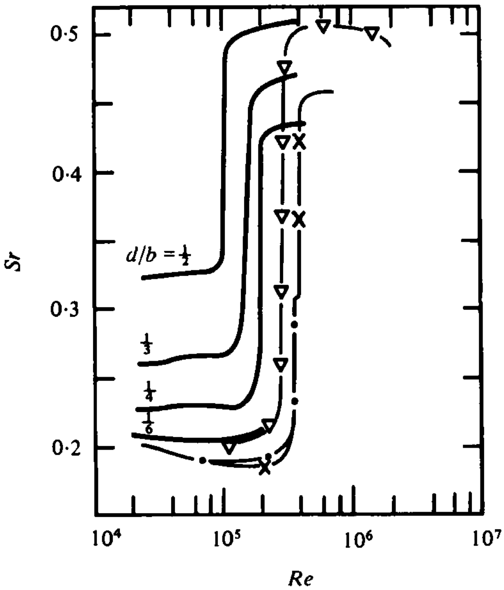

FIGURE 11. Effect of blockage ratio on Strouhal number after Richter (1973). ——, Richter (1973); —·—, Drescher (1956); —×—, Bearmann (1969); —▽—, present.

which is close to the critical Mach number for cylinders ($Ma = 0{\cdot}44$) where the velocity of sound is already locally reached near the main span of the cylinder. Perhaps their results are influenced by compressibility effects.

The influence of large blockage ratios, d/b, on the vortex-shedding mechanism has been investigated by Richter (1973) and Richter & Nandascher (1976). He used a cylinder with an aspect ratio of 8·62. The relative roughness was $k/d = 200 \times 10^{-5}$, and the blockage ratio was varied in the range $1/2 > d/b > 1/6$. Richter's results are plotted together with those of Drescher (1956), Bearman (1969), and the authors' in figure 11 for different blockage ratios. Reference velocity is that of the incident flow. The Strouhal number rises with growing blockage ratio because the velocity at the separation point increases compared to unconfined flow conditions. For the same reason the critical Reynolds number is shifted to smaller values with increasing d/b.

In the subcritical range the present results ($d/b = 1/6$) fit very well into Richter's curve for $d/b = 1/6$. Also Drescher's results ($d/b \rightarrow 0$) and those of Bearman ($d/b = 1/15{\cdot}4$) are in good agreement. For critical flow conditions Richter measured increasing values of Sr with increasing blockage ratio. The absolute values, however, are lower than those expected with respect to Bearman's and the present findings. As the turbulence level was almost equal in all tests mentioned, this departure is most probably due to the surface roughness of Richter's test cylinder ($k/d = 200 \times 10^{-5}$). Unfortunately Richter did not mention whether the surface roughness was determined for tangential or axial direction. It is likely that the rather high roughness of $k/d = 200 \times 10^{-5}$ resulting from the turning and polishing process during construction was measured along the cylinder. Furthermore, this value seems to be the ratio of maximum roughness height to diameter. Therefore the equivalent sand-grain parameter will have a lower value. This may give an explanation for the evidence that Richter's data of the Strouhal number at critical conditions are considerably higher than the present values measured for a roughness parameter of $k_s/d = 300 \times 10^{-5}$.

4.2. *Rough cylinders*

Szechenyi (1975) has published experimental data on vortex shedding from artificially roughened cylinders for $9{\cdot}6 \times 10^4 < Re < 6{\cdot}5 \times 10^6$ and for a roughness parameter of $15 \times 10^{-5} < k/d < 200 \times 10^{-5}$. The roughness was produced by gluing glass beads of diameter δ to the surface of cylinders. Szechenyi defined the relative roughness as the ratio of δ to the diameter d of the cylinder. This is different from the authors' definition, which is based on Achenbach's (1971) result that when using spheres as roughness elements the equivalent sand-grain roughness is $k_s = 0{\cdot}55 \times \delta$.

According to Szechenyi's findings the upper limit of k_s/d (present definition) below which regular shedding occurred was $k_s/d = 120 \times 10^{-5}$ whereas for $k_s/d = 150 \times 10^{-5}$ no regular shedding was observed at a Reynolds number of $3{\cdot}3 \times 10^6$. This is in contradiction to the present results which show that the regularity of vortex shedding is not affected by increasing roughness in the whole range of roughness parameter examined.

Teverovskii (1968) investigated the influence of single roughness elements (0·05–0·4 mm high, 0·8 mm wide) on the shedding frequency. He fastened these elements on a smooth cylinder along a generator line at various angles of circumference. Unfortunately a detailed description of the arrangement is not given. The relative roughness k/d ranged from 500×10^{-5} to 2000×10^{-5}. The Reynolds number was varied from $1{\cdot}5 \times 10^4$ to $2{\cdot}5 \times 10^5$. For two Reynolds numbers ($9{\cdot}2 \times 10^4$, $1{\cdot}37 \times 10^5$) results are reported. Depending on k/d and the position of the roughness elements the Strouhal number is lower or higher than that of the smooth cylinder, the limits being $Sr = 0{\cdot}21$ and $Sr = 0{\cdot}15$. The conclusion which can be drawn from these rather sparse results is that individually distributed roughness elements may have an effect on the Strouhal number different from that of uniformly distributed roughnesses.

This influence depends on whether the individual roughness causes final boundary-layer separation or not.

5. Summary

The non-steady wake flow behind cylinders with smooth and rough surfaces has been investigated. Different relative roughnesses ($k_s/d = 75 \times 10^{-5}$, 300×10^{-5}, 900×10^{-5}, 3000×10^{-5}) have been tested. The Reynolds number varied from 6×10^3 to $4 \cdot 5 \times 10^6$. In the case of the smooth cylinder, experimental results of other authors are in good agreement with the authors' findings. In the critical and supercritical flow regime the regular vortex shedding from the cylinder with low length-to-diameter ratio, $l/d = 3 \cdot 38$, was suppressed because of the three-dimensional wake flow.

For rough cylinders too, a sudden increase of the Strouhal number is observed at critical flow conditions. With increasing roughness parameter this step becomes smaller compared with the smooth cylinder. In the transcritical flow range the Strouhal number varies only by $\pm 7\%$ from the mean value for all roughness parameters tested.

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